# Long distance contribution to $K^+ \to \pi^+ \nu \bar{\nu}$ decay and $O(p^4)$ terms in CHPT

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## **ABSTRACT**

We calculate the long distance contribution to  $K^+ \to \pi^+ \nu \bar{\nu}$  using chiral perturbation theory. The leading contribution comes from  $O(p^4)$  tree terms. The branching ratio of the  $O(p^4)$  long distance contribution is found to be of order  $10^{-3}$  smaller than the short distance contribution.

#### 1 Introduction

Chiral perturbation theory (CHPT) has been applied in the analysis of long distance effects in the  $K^+ \to \pi^+ \nu \bar{\nu}$  decay [1, 2]. The observation of  $K^+ \to \pi^+ \nu \bar{\nu}$  decay seems possible in the near future [3]. An upper limit on the branching ratio for the  $K^+ \to \pi^+ \nu \bar{\nu}$  decay has been set at  $2.4 \times 10^{-9}$  (90%C.L.) [4]. The short distance loop diagrams dominate, due to the explicit dependence of the heavy top quark mass [5]. Sizable contributions come also from internal charm quark exchanges [5, 6]. The QCD corrections to this decay have been calculated in the leading logaritmic approximatation [6, 7, 8]. Calculated next-to-leading QCD corrections reduced considerably the theoretical uncertainty due to the ambiguity in the choice of the renormalization scale present in the leading order expression [9, 10, 11].

The  $K^+(k) \to \pi^+(p)\nu(p_{\nu})\bar{\nu}(p_{\bar{\nu}})$  decay amplitude can be written in the form

$$\mathcal{M}(K^{+} \to \pi^{+} \nu \bar{\nu}) = \frac{G_{F}}{\sqrt{2}} \frac{\alpha f_{+}}{2\pi sin^{2} \theta_{W}} [V_{ts}^{*} V_{td} \xi_{t}(m_{t}^{2}/M_{W}^{2}) + V_{cs}^{*} V_{cd} \xi_{c}(m_{c}^{2}/M_{W}^{2}) + V_{us}^{*} V_{ud} \xi_{LD}] (k+p)^{\mu} \bar{u}(p_{\nu}) \gamma_{\mu} (1-\gamma_{5}) v(p_{\bar{\nu}}), \quad (1)$$

where  $G_F$  denotes the Fermi constant,  $\theta_W$  is the weak mixing angle,  $\alpha$  is the fine structure constant,  $f_+$  is the form factor in  $\bar{K}^0 \to \pi^+ e \bar{\nu}_e$  decay and  $V_{ij}$  stands for  $i \to j$  element of the CKM matrix. The functions  $\xi(x_q)$  arise from the short distance contribution to  $K^+ \to \pi^+ \nu \bar{\nu}$  decay [6, 9] and  $\xi_{LD}$  from the long distance contribution [1, 2, 12, 13].

In this paper we concentrate on the long distance contribution to the part of amplitude  $\xi_{LD}$ , that arises from the time ordered product of the weak  $\Delta S = 1$  effective Hamiltonian with the  $Z^0$  neutral current. Due to the  $\Delta I = 1/2$  rule [1, 12], this contribution dominates the one coming from Feynman diagrams with two W bosons.

In CHPT the  $O(p^2)$  tree level  $K^+ \to \pi^+ Z^0 \to \pi^+ \nu \bar{\nu}$  decay amplitude vanishes [1, 2]. The loop contributions were calculated in [2], leading to the branching ratio  $BR(K^+ \to \pi^+ \nu \bar{\nu}) = 7.7 \times 10^{-18}$ , roughly of order  $10^{-7}$  smaller than that of the short distance contribution [6].

We determine the long distance contribution, using the  $O(p^4)$  strong and weak Lagrangians of CHPT. The number of counterterms in the effective weak Lagrangian of  $O(p^4)$  is very large [14, 15, 16]. However, it is possible to

reduce them, using predictions for the weak Lagrangian in specific models [14-23]. We construct the weak Lagrangian of  $O(p^4)$  relying on the factorization approach.

The paper is organized as follows: In Sect. 2 we discuss effective strong and weak Lagrangians of  $O(p^4)$ . Sect 3. contains the long distance contribution to the  $K^+ \to \pi^+ \nu \bar{\nu}$  decay amplitude and the branching ratio. A short conclusion is presented in Sect. 4.

### **2** Effective Lagrangians at $O(p^4)$ in CHPT

At the lowest order in momentum  $O(p^2)$  the strong chiral Lagrangian is given by

$$\mathcal{L}_s^2 = \frac{f^2}{4} \{ Tr(D_\mu U^\dagger D^\mu U) + Tr(\chi U^\dagger + U \chi^\dagger) \}, \tag{2}$$

where  $U = -\frac{i\sqrt{2}}{f}\phi$ ,  $f \simeq f_{\pi} = 0.093$  GeV is a pion decay constant and  $\phi$  is a pseudoscalar meson matrix (see eg. [17, 18, 24]). The covariant derivative is given by  $D_{\mu}U = \partial_{\mu}U + iUl_{\mu} - ir_{\mu}U$ ,  $l_{\mu}$  and  $r_{\mu}$  are external gauge field sources. The explicit chiral symmetry breaking induced by the electroweak currents of the standard model corresponds to the following choice:

$$r_{\mu} = eQ[A_{\mu} - tan\theta_W Z_{\mu}] \tag{3}$$

$$l_{\mu} = eQ[A_{\mu} - tan\theta_{W}Z_{\mu}] + \frac{e}{sin\theta_{W}}Q_{L}^{(3)}Z_{\mu} + \frac{e}{\sqrt{2}sin\theta_{W}}[Q_{L}^{(+)}W_{\mu}^{(+)} + Q_{L}^{(-)}W_{\mu}^{(-)}].$$
(4)

Here the Q's are the electroweak matrices

$$Q = \frac{1}{3}diag(2, -1, -1), \quad Q_L^{(3)} = \frac{1}{2}diag(1, -1, -1), \tag{5}$$

$$Q_L^{(+)} = (Q_L^{(-)})^{\dagger} = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$
 (6)

The matrix  $\chi = 2B[s(x) + ip(x)]$ , with  $s + ip = \mathcal{M} = diag(m_u, m_d, m_s)$ , takes into account the explicit breaking due to the quark masses in the underlying QCD Lagrangian [24, 25]. The matrix  $\chi$  can be fixed by physical pseudoscalar masses to lowest order in the chiral expansion. The Lagrangian of  $O(p^4)$  order in CHPT [24, 25] is given by

$$\mathcal{L}_{s}^{4} = L_{1}(Tr(D_{\mu}U^{\dagger}D^{\mu}U))^{2} + L_{2}Tr(D_{\mu}U^{\dagger}D_{\nu}U)Tr(D^{\mu}U^{\dagger}D^{\nu}U) 
+ L_{3}Tr(D_{\mu}U^{\dagger}D^{\mu}UD_{\nu}U^{\dagger}D^{\nu}U) + L_{4}Tr(D_{\mu}U^{\dagger}D^{\mu}U)Tr(\chi U^{\dagger} + U\chi^{\dagger}) 
+ L_{5}Tr(D_{\mu}U^{\dagger}D^{\mu}U(\chi U^{\dagger} + U\chi^{\dagger})) + L_{6}(Tr(\chi U^{\dagger} + U\chi^{\dagger}))^{2} 
+ L_{7}(Tr(\chi^{\dagger}U - \chi U^{\dagger}))^{2} + L_{8}Tr(\chi^{\dagger}U\chi^{\dagger}U + \chi U^{\dagger}\chi U^{\dagger})) 
-i L_{9}Tr(F_{R}^{\mu\nu}D_{\mu}UD_{\nu}U^{\dagger} + F_{L}^{\mu\nu}D_{\mu}U^{\dagger}D_{\nu}U) + L_{10}Tr(U^{\dagger}F_{R}^{\mu\nu}UF_{L\mu\nu}) 
+ H_{1}Tr(F_{R\mu\nu}F_{R}^{\mu\nu} + F_{L\mu\nu}F_{L}^{\mu\nu}) + H_{2}Tr(\chi^{\dagger}\chi) \tag{7}$$

where  $F_L^{\mu\nu} = \partial^{\mu}l^{\nu} - \partial^{\nu}l^{\mu} - i[l^{\mu}, l^{\nu}]$  and  $F_R^{\mu\nu} = \partial^{\mu}r^{\nu} - \partial^{\nu}r^{\mu} - i[r^{\mu}, r^{\nu}]$  and  $L_1, \ldots L_{10}$  are ten real low-energy coupling constants, which completely determine the low-energy behaviour of pseudoscalar meson interactions to  $O(p^4)$  ( $H_1$ ,  $H_2$  are of no physical significance). This  $O(p^4)$  Lagrangian contains all possible terms which are allowed by chiral invariance. All the one loop divergences, which by power counting can only give rise to local  $O(p^4)$  terms, are absorbed by suitable renormalizations of the  $L_i$  and  $H_{1,2}$  constants, as it was performed in [24]. Generally  $L_i$  are divergent (except  $L_3$ , and  $L_7$ ) and they depend on a renormalization scale  $\mu$ , which is not seen in observable quantities. The  $H_i^r$  can be determined, although they are not accessible experimentally. In fact, once the  $O(p^4)$  strong chiral Lagrangian is chosen in the form of [24, 25], these couplings can be fixed by resonance exchange.

The authors of [25] calculated contributions of all low-lying resonances to the renormalized couplings  $L_i^r$ . Using the equations of motion for resonances, they found that the  $L_i^r$ , at the scale  $\mu = M_\rho$ , get contributions from vector, axial vector and scalar resonances. In the Table 1 we give the numerical values of the  $L_i^r$  and  $H_1^r$ , which we use later in our calculations. The Model I in the Table 1 denotes the  $L_i^r$  and  $H_1^r$  from [25].

In our numerical calculations we use the values of the coupling constants  $L_i^r$  and  $H_i^r$  derived within the extended Nambu and Jona-Lasinio model [26]. We concentrate on the fit 1 and the fit 4 of [26]. The reasons are following: the fit 1 is obtained for the most favorable set of parameters required by the extended Nambu and Jona Lasinio model [26], while the fit 4 was quite

successful in explaining the experimental result for the  $K^+ \to \pi \gamma^* \to \pi e^+ e^-$  weak counterterm [23]. The Model II (the fit 1 and the fit 4) in the Table 1 refers to the  $L_i^r$  and  $H_1^r$  from [26].

The weak Lagrangian of  ${\cal O}(p^2)$  in the chiral expansion has a unique form [14-18]

$$\mathcal{L}_w^2 = G_8 f^4 Tr(\lambda_6 D_\mu U^\dagger D^\mu U). \tag{8}$$

The chiral coupling  $G_8$  can be expressed using the well known constants in the Standard model  $G_8 = \sqrt{\frac{1}{2}}G_F V_{ud}V_{us}^* g_8$ . The coupling  $|g_8| = 5.1$  was found analysing  $K \to \pi\pi$  decay [14, 16, 20].

The charged weak current to lowest order in chiral perturbation theory can be derived as [14, 20, 21]

$$J_{\mu}^{(1)}|_{ij} = \frac{\delta S_2}{\delta(l_{\mu})_{ij}}.$$
 (9)

 $S_2$  is the action determined by  $\mathcal{L}_s^2$  in (2), indeces i, j denote u, d, s flavor. The factorization model results in writing the weak Lagrangian (8) as [14]

$$\mathcal{L}_{w}^{2} = 4G_{8}Tr(\lambda_{6}J_{\mu}^{(1)}J^{(1)\mu}). \tag{10}$$

The generalization on higher order terms in momentum leads to [14, 15, 16, 20, 21]

$$J_{\mu}|_{ij} = J_{\mu}^{(1)}|_{ij} + J_{\mu}^{(3)}|_{ij} + \cdots$$

$$= \frac{\delta S_2}{\delta(l_{\mu})_{ij}} + \frac{\delta S_4}{\delta(l_{\mu})_{ij}} + \cdots,$$
(11)

where  $S_4$  stands for the effective action at given order in momentum. The effective weak Lagrangian is then

$$\mathcal{L}_w = 4G_8 Tr(\lambda_6 J_\mu J^\mu). \tag{12}$$

There are other approaches used to construct the weak Lagrangian at  $O(p^4)$  [14, 17, 18, 19, 22]. A certain confidence in the applicability of the factorization approach, we gain from the analysis of the  $K^+ \to \pi^+ \gamma^* \to \pi^+ e^+ e^-$  decay, which branching ratio is measured. In this decay, the finite part of

the long distance contribution seems to be better described within the factorization approach than within the weak deformation model [23].

## 3 $K^+ \to \pi^+ \nu \nu$ decay and $O(p^4)$ contributions

We write the  $K^+(k) \to \pi^+(p)\nu(p_\nu)\bar{\nu}(p_{\bar{\nu}})$  decay amplitude in the following form

$$M(K^+ \to \pi^+ \nu \bar{\nu}) = \frac{e}{4sin\theta_W cos\theta_W} M_\mu \bar{u}(p_\nu) \gamma_\mu (1 - \gamma_5) v(p_{\bar{\nu}}) \qquad (13)$$

with

$$M_{\mu} = g_{+}(q^{2})(k+p)_{\mu} + g_{-}(q^{2})(k-p)_{\mu}$$
 (14)

The component along the momentum transfer  $q_{\mu} = (k - p)_{\mu}$  gives no contribution to the unpolarized decay width. At the leading order  $O(p^2)$  the sum of the pole and direct weak transition vanishes [1, 2].

Using the strong Lagrangian given in (7), and the weak Lagrangian of  $O(p^2)$  from (8), we obtain "indirect" (pole contribution) for the  $K^+(k) \to \pi^+(p)Z^0(\epsilon,q)$  decay, Fig. 1 (a) :

$$<\pi^{+}Z^{0}|\mathcal{L}_{pole}|K^{+}> = <\pi^{+}Z^{0}|\mathcal{L}_{s}^{4}|\pi^{+}> \frac{-1}{m_{K}^{2}-m_{\pi}^{2}} <\pi^{+}|\mathcal{L}_{w}^{2}|K^{+}>$$

$$+ <\pi^{+}|\mathcal{L}_{w}^{2}|K^{+}> \frac{-1}{m_{\pi}^{2}-m_{K}^{2}} < K^{+}Z^{0}|\mathcal{L}_{s}^{4}|K^{+} > (15)$$

The direct weak transition is present also, Fig. 1 (b)

$$<\pi^{+}Z^{0}|\mathcal{L}_{w}^{4}|K^{+}> = 4G_{8} < \pi^{+}Z^{0}|Tr(\lambda_{6}\{J_{\mu}^{(1)}, J^{(3)\mu}\})|K^{+}>$$
 (16)

with the weak current calculated using (11). Neglecting terms of order  $m_{\pi}^2/m_K^2$  in the decay amplitude  $K^+(k) \to \pi^+(p)Z^0(\epsilon,q)$ , we obtain for the indirect (pole) transition of  $O(p^4)$ 

$$<\pi^{+}Z^{0}|\mathcal{L}_{pole}|K^{+}> = 4iG_{8}\epsilon \cdot (k+p)(L_{4}^{r}8m_{K}^{2} + L_{9}^{r}q^{2})(s-t)$$
 (17)

and for the direct weak transition of  $O(p^4)$ 

$$<\pi^{+}Z^{0}|\mathcal{L}_{dir}|K^{+}> = 4iG_{8}\epsilon \cdot (k+p)[-L_{4}^{r}16(s-t)m_{K}^{2} + L_{5}^{r}4(t-2s)m_{K}^{2} + L_{9}^{r}2(t-s)q^{2} + L_{10}^{r}\frac{2}{3}tq^{2} + H_{1}^{r}(\frac{4}{3}t-4s)q^{2}].$$
 (18)

We use the notation

$$s \equiv \frac{ie}{2sin\theta_W cos\theta_W}, \ t \equiv ietan\theta_W.$$
 (19)

The authors of [2] have calculated the finite part of the one loop amplitude. They found that there is a divergent part of  $O(p^4)$ . There is a standard procedure used to cancel out divergent contributions coming from the one loop effects of  $O(p^4)$  [17, 18, 24]: one constructs the  $O(p^4)$  counterterms in such a way, that divergences are reabsorbed in a renormalization of the various coupling constants. Therefore, we assume that the divergences coming from the loops of  $O(p^4)$  [2] are cancelled out by divergent parts of the couplings of  $O(p^4)$ . It means that the contributions calculated in (17) and (18) determine the finite part of the cunterterms of  $O(p^4)$ . The  $g_+^r(q^2)$  form factor is found from (17) and (18) to be

$$g_{+}^{r}(q^{2}) = \frac{2eG_{8}}{m_{Z}^{2}sin\theta_{W}cos\theta_{W}} \{8m_{K}^{2}L_{4}^{r}(2sin^{2}\theta_{W} - 1) + 8m_{K}^{2}L_{5}^{r}(sin^{2}\theta_{W} - 1) + q^{2}[L_{9}^{r}(2sin^{2}\theta_{W} - 1) + \frac{4}{3}L_{10}^{r}sin^{2}\theta_{W} + H_{1}^{r}(\frac{8}{3}sin^{2}\theta_{W} - 4)]\}. \quad (20)$$

We neglect the logarithmic piece, since it was found to be very small [2].

The size of the long distance contribution  $\xi_{LD}$  defined in (1), can be estimated by taking  $q^2=0$  in the form factor  $g_+^r(q^2)$  in (20), and  $f_+(0)=1$  in (1). We obtain the amplitude  $\xi_{LD}(0)=-2.6\times 10^{-5}$ , using the couplings  $L_i^r$  and  $H_1^r$  from the Table 1 (Model I). With  $L_i^r$  and  $H_1^r$  found in the fit 1 (Model II, Table 1), the amplitude becomes  $\xi_{LD}(0)=-3.0\times 10^{-5}$ . With the couplings from the fit 4 (Model II, Table 1), the amplitude obtains the largest value  $\xi_{LD}(0)=-3.2\times 10^{-5}$ . These results are in agreement with the estimation made by Lu and Wise [1]. They have noticed that the long distance contribution to  $K^+ \to \pi^+ Z^0 \to \pi^+ \nu \bar{\nu}$  in CHPT might be of order  $10^{-5}$ . This should be compared with the charm quark short distance contribution  $\xi_c \simeq 10^{-3}$ .

The form factor  $g_+^r(q^2)$  in (20) helps to calculate the decay width for  $K^+ \to \pi^+ \nu \bar{\nu}$ 

$$\Gamma(K^+ \to \pi^+ \nu \bar{\nu}) = C \int_{m_{\pi}}^{\frac{m_K^2 + m_{\pi}^2}{2m_K}} |g_+^r(q^2)|^2 (E_{\pi}^2 - m_{\pi}^2)^{\frac{3}{2}} dE_{\pi}, \qquad (21)$$

with

$$C = \frac{\alpha m_K}{24\pi sin^2 \theta_W cos^2 \theta_W}. (22)$$

Using the numerical values of  $L_i^r H_1^r$  from Table 1 (Model I), we calculate the branching ratio  $BR(K^+ \to \pi^+ \nu \bar{\nu})_{LD} = 0.17 \times 10^{-13}$ .

The couplings  $L_i^r$ , obtained in the fit 1 (Model II, Table 1), lead to  $BR(K^+ \to \pi^+ \nu \bar{\nu})_{LD} = 0.29 \times 10^{-13}$ , and the couplings  $L_i^r$ , from the fit 4 (Model II, Table 1) give the largest, among calculated, branching ratio

$$BR(K^+ \to \pi^+ \nu \bar{\nu})_{LD} = 0.40 \times 10^{-13}.$$
 (23)

For comparison, we mention that the branching ratio coming from the short distance contribution is very close to  $10^{-10}$  [9, 10, 27].

#### 4 Concluding Remarks

The long distance contribution to the  $K^+ \to \pi^+ Z^0 \to \pi^+ \nu \bar{\nu}$  decay is calculated using the  $O(p^4)$  tree level of CHPT.

The amplitude and the branching ratio depend on the model used for the  $O(p^4)$  couplings  $L_i$  and  $H_i$  (the resonance exchange model [25] or the extended Nambu and Jona-Lasinio model [26]).

The branching ratio can be as large as  $BR(K^+ \to \pi^+ Z^0 \to \pi^+ \nu \bar{\nu})_{LD} = 0.4 \times 10^{-13}$ . This is still too small in comparison with the dominant top and charm quark short distance contribution. Therefore, we support the suggestion of [1, 3, 9, 10, 11] that with the known mass of top quark a measurement of the branching ratio  $BR(K^+ \to \pi^+ \nu \bar{\nu})$  would lead to a precise determination of the mixing angle  $V_{td}$ .

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# Figure Caption

Fig. 1. Feynman diagrams which contribute to the  $K^+ \to \pi^+ Z^0$  vertex in CHPT. The square detones the weak interaction vertex of  $O(p^2)$ . The crossed circle (square) stand for the  $O(p^4)$  strong (weak) vertex.

$O(p^4)$ coup.	Model I	$Model \ II(fit \ 1)$	Model II(fit 4)
$L_5^r$	$1.4 \times 10^{-3}$	$1.6 \times 10^{-3}$	$1.7 \times 10^{-3}$
$L_9^r$	$6.9 \times 10^{-3}$	$7.0 \times 10^{-3}$	$7.1 \times 10^{-3}$
$L_{10}^{r}$	$-6.0 \times 10^{-3}$	$-5.9 \times 10^{-3}$	$-5.1 \times 10^{-3}$
$H_1^r$	$-7.0 \times 10^{-3}$	$-4.7 \times 10^{-3}$	$-2.4 \times 10^{-3}$

Table 1: The  $L_i^r$  i=5,9,10 and  $H_1^r$  denote the  $O(p^4)$  couplings calculated in the resonance exchange model of [25] (Model I) and the extended Nambu and Jona Lasinio model of [26] (Model II). The couplings coming from the fit 1 (fit 4) in [26] are presented in the second (third) column. (The  $L_4^r=0$  in both models.)

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